Abstract

The aim of this paper is to present and develop mathematical models optimizing all materials flows in supply chain. It is so important for every manager to reduce their costs and have optimum level stock to not facing the overhead costs. In this paper a supply chain network of car part producer is conducted and a mathematical model is proposed. The functions consist of all costs like, transportation cost, production cost. In the first part of research the literature review and the next part, problem definition is discussed. In this paper a 4 echelons supply chain studied and for every chain a mathematical model is proposed. Every material which entering to one of systems, prepared to enter for the next level and this continues until products achieve the markets. In this study, we showed 5 case studies and numerical examples to analyze the model. The proposed models are more reliable and could be used as DSS for managers. The findings also depict the different status of model, and decision makers could make an appropriate decision. The novelty of this paper is that team work flexibility and machines flexibility is entered to model.

Keywords: Mathematical Model, Supply Chain Network, Flexibility, Optimizations.
1- Introduction

According to rapid development of business world, information and manufacture technology, the demand of consumers, production and sale all have changed. All industrial and service-based organizations begin to develop internationally. Today, most of companies attend to this point that effective supply chain management can be one of the most important instruments to increase competitive advantage. Therefore, creation of subfigures and stable relationships among suppliers and buyers as well as being success in selecting supplier can be the main factor for the success of supply chain (Zandehessami et al, 2011). In this era of global competition, the enterprises could achieve the successfulness that have a tight integration between resources on supply chain and all factories, respond to customers’ requirements efficiently, provide products of higher quality, reduce the operation cost and raise the satisfaction of customers are the key points to make an enterprise have international competence (Ying-Hua et al, 2008).

Assessment and selection of suppliers play the important role in supply chain and is very necessary for the success of industrial organization (Hartley and Choi, 1996; Deagraeve et al, 2000). This issue is important because sold goods and services have direct relation with bought materials and services. Traditionally, suppliers are selected among those whom have ability to represent concerned quality, time of delivery and suggestive price (Sevkli et al, 2007).

With such process, sellers will offensively compete. Relations among buyers and sellers are completely competitive. In the traditional process of buying, special attention has been paid to financial operations among buyers and sellers. In such process, principle aim of buyers is the lowest price so that to make a powerful competition among suppliers and discussion with them.
However, in today modern organizations, most of them prefer to use new suppliers' strategy (Chandra and Kumar, 2000).

In fact, this strategy means that buyer can have long time relations and cooperation with limited numbers of suppliers. Because failing suppliers brings about reduction of production costs and financial recreations (Koh et al., 2007). Cooperation among buyers and suppliers is the initial point for establishing successful serial supply. This matter is very necessary but is not enough. In the next phase, we need cooperation and coordination among buyers and suppliers. This cooperation include flow of special working, sharing information through electronically communication of data and internet, common programming and other mechanisms organizing production systems of Just In Time and Total Quality Management (Spekman et al., 1998).

The matter of selecting supplier usually involves more than one criterion and criteria are often against each other (Yang et al 2008). Hence, in order to select supplier, we need an instrument which can perform this important case regarding multi criteria. Extra to selection of suitable supplier, theoretical methods of decision making can be used in order to assess relations of buyer and supplier. In the matter of multi criterion decision making, some of alternatives regarding several indexes were analyzed and its output is prioritization among them.

Different factors are used as the criterion of selecting supplier being: price, approach of just in time delivery, known in industry, size of organization, geographical location, and quality, evaluation of environment, capacity, services, and delay in delivering good, packing, transportation and storing.

Supply chain management (SCM) has been a hot topic in the management arena in the recent years. The term “supply chain” (SC) conjures up images of products, or supplies, moving from manufacturers to distributors to retailers to customers, along a chain, in order to fulfill a
customer request (Gong et al. 2008). Supply chain management includes unification of supply chains activities and information flows related them through improvement in relations of chain in order to achieve unfailing competitive benefits. Thus supply chain management include process of unifying activities of supply chain and information flows related them through improvement and unification of activities in supply chain of production and supply of goods (Hull et al 1999).

With increase of competition in the global field, companies should focus on speed and carefulness in answering customer's need and deliberate for customer's satisfaction. This process has been attended in the supply chain. Performed attempts in supply chain reduce risk and uncertainty and improve representation of service to customer, improvement of stock levels, processes of business and time cycles and finally bring about increase of competition, customer's satisfaction and benefits (Chou & Chang 2008).

Supply chain management is process of programming, administration and control of operating supply chain with operational methods (Stevenson 2002). Limit of supply chain management involves transposition, stock of raw materials, stock of producing goods and produced goods from the primary point to the consumed point. Figure 1 shows a pattern of supply chain in a factory.

**Figure 1 about here**

The most important members of supply chain are suppliers; producer and customers that supply chain management follow to establish relations among these members. Communications in supply chain are performed by several ways. In fact, these relations are related to three types of flow in supply chain (Stevenson 2002).

1) **Physical flow of materials:**
Physical flow of materials is movement of raw materials, done processing on raw materials and its movement by distributors towards final customers. In this type of flow, physical movement of materials is from high level to low level.

2) **Flow of information or communication of information:**

In this type of flow, information is transacted among members of supply chain. Movement of information in supply chain is bilateral, meaning that it is fulfilled from high level to low level and vice versa.

3) **Financial flow or cash flow:**

Creation of financial flows is the principle motivation in creating supply chain. Movement of financial flow is usually done from low level to high level.

During current periods, usage of supply chain has had many benefits for organization that we refer to the following cases:

1) Reduction of stock;
2) Reduction of production costs;
3) Productivity increcent;
4) More agility;
4) Orders time reduction;
5) Customer's more honesty.

One of important tasks in supply chain is decision making in procurement or producing. There are six important processes in decision making about procurement or producing that are:

1. Production or purchase,
2. Selection of supplier,
3. Contract and discussion,
4. Designing cooperation,
5. Procurement,
6. Sourcing analysis.

Process of procurement is shown in figure 2.

Figure 2 about here

2- Literature Review

There is plethora of research conducted to establish an appropriate supply chain in order to increase the efficiency and reduce cost of it. To cope with the complicated problem of supply chain, researchers have used mathematical programming or heuristics algorithm to solve such problems. The challenge in global SCM is the development of decision-making frameworks that accommodate diverse concerns of multiple entities across the supply chain. Considerable efforts have been expended in developing decision models for SC problems (Narasimhan & Mahapatra, 2004). Melachrinoudis, et al. (2000) applied multi objective and integer programming for two-echelon plant/warehousing facility to solve the problem. Ross (2000) implemented performance-based strategic resource allocation to solve the supply chain network design. Syam, (2002) used meta heuristic method called simulated annealing and Lagrangian relaxation in supply chain network environment of multiple echelons to minimize the costs. Schwarz (1973) applied one-warehouse N-retailer inventory model. Muckstadt and Roundy (1987) assessed multi-item, one-warehouse, multi-retailer in a distribution system. Cheshmberah et. al (2011) proposed a mathematical model to optimize single-commodity distribution in chain stores network. Yang and Wee (2000) proposed a model for buyer and vendor by differential equations. Banerjee


(1986) for the first time, present the two-echelon inventory model for vendor and buyer. Hill (1999) presented a production and inventory model with integration as a whole. Lau and Lau (2001) proposed a two-echelon problem where the retailer has better market information than the manufacturer. Pasternack (2001) applied a single inventory problem vendor (retailer) has limited funds to purchase items to sell.

The issues of how to simultaneously integrate manufacturing and distribution systems in a supply chain with multi objectives have attracted considerable interest from both practitioners and academics (Liang and Cheng, 2009) respectively. Cohen and Lee (1989) present a deterministic, mixed integer, non-linear programming with economic order quantity technique to develop a global supply chain plan. Output of the model provides global resource deployment policy for the plants, distribution centers and customer zones. Turan Paksoy et al (2010) applied fuzzy nonlinear multi-objective mathematical model for supply chain network.

time period production/distribution planning decisions problems with fuzzy objectives. Liang and Cheng (2009) apply fuzzy sets to multi-objective manufacturing/distribution planning decision problems with multi-product and multi-time period in supply chains by considering time value of money for each of the operating categories. Peidro et al. (2009) propose a new mathematical programming model for supply chain planning under supply, process and demand uncertainty. The model has been formulated as a fuzzy mixed integer linear programming model where data are ill-known and modeled by triangular fuzzy numbers. Cao et al. (2010) develop stochastic chance constrained mixed-integer nonlinear programming models to solve the refinery short-term crude oil scheduling problem. Zandhessami et al (2011) proposed a hybrid ANP and fuzzy goal programming to select the best supplier.

Also Aprile et. al (2005) proposed a model considering the flexibility of supplier and producer.

Their model is as below:

$$\text{max } z = \sum_{i}^{M} \sum_{j}^{N} \sum_{p}^{P} (x_{ij}D_{pi})$$

S.t. $\sum_{i}^{P} \sum_{j}^{M} (x_{ij}D_{pi}) \leq (A\tilde{C}_j) \quad \forall j$ (a)

$$\sum_{i}^{M} (x_{ij}D_{pi}) - \sum_{k}^{K} (x_{kj}S\tilde{C}_k) = 0 \quad \forall j, \forall p$$ (b)

$$\sum_{i}^{M} (x_{ij}) \leq 1 \quad \forall i, \forall p$$ (c)

$$\sum_{p}^{P} \sum_{j}^{N} (x_{kj}) \leq 1 \quad \forall k$$ (d)

$$(x_{ij}, x_{kj}) \geq 0 \quad \forall i, \forall j, \forall k, \forall p$$ (f)
In their model, the function is maximizing production in time horizon. AC\textsubscript{j} is total capacity of j\textsuperscript{th} producers. Constraint “a” ensures that the allocated demand to producers would not be more than their capacity. Constraint “b” satisfies the supply and demand in the model. Also, constraints “c”, “d” and “f” are necessary to obtain the feasible results (Aprile et. al , 2005).

In fact, in this paper, we want to develop this model to achieve the more reliable results. The next step we propose the developed model.

3- Problem definition

This study is implemented in one of Iranian car part manufacture. This is multi echelon and multi products supply chain network model. The echelon divided in four parts: supplier level 1, supplier level 2, Factory and Market.

Figure 3 is showing the supply chain network of this research.

Figure 3 about here

In this part, supplier 1 in modeled to satisfy the second level supplier. The materials which are produced in first level will transfer to second level. First level supplier uses all facilities to respond and satisfy the next supplier demand. Indices used in models are as below:

- i: sign of product (i = 1,2,..,I)
- a: sign of factory (a = 1,2,..,A)
- h: sign of market (h = 1,2,..,H)
- j: sign of team works in factory
- \( b \): sign of goods produced in level 2 (\( b = 1,2,\ldots,B \))
- \( z \): sign of second level supplier (\( z = 1,2,\ldots,Z \))
- \( e \): sign of team works in second level supplier (\( e = 1,2,\ldots,E \))
- \( c \): sign of machines work in second level supplier (\( c = 1,2,\ldots,C \))
- \( f \): sign of raw materials produced by first level supplier (\( f = 1,2,\ldots,F \))
- \( g \): sign of first level supplier (\( g = 1,2,\ldots,G \))
- \( k \): sign of team works in first level supplier (\( k = 1,2,\ldots,K \))
- \( q \): sign of machines work in first level supplier (\( q = 1,2,\ldots,Q \))

So the model is introduced for the first level as below:

\[
\begin{align*}
\min z \quad & \sum_{z} \sum_{i} \sum_{g} TR_{gz} X_{fgz} + \sum_{z} \sum_{f} \sum_{g} PC_{fg} X_{fgz} + \sum_{z} \sum_{g} \eta_{g} \cdot TR_{gz} \\
\text{s.t.} \quad & \sum_{f} \left( \sum_{z} X_{fkgz} \right) \alpha_{kg} + d_{k-} - d_{k+} = T_{kg} \quad \forall k, \forall g \\
& \sum_{f} \left( \sum_{z} X_{fkgz} \right) \delta_{kg} + S_{fgz} = T_{fkg} \quad \forall g, \forall q \\
& \sum_{z} \sum_{g} \left( \sum_{f} \left( X_{fkgz} + X_{fkgz} \right) \right) = 0.5 \sum_{z} \sum_{g} \sum_{f} X_{fgz} \quad \forall k, \forall q \\
& \sum_{z} \sum_{g} X_{fgz} = b \left( \sum_{a} \left( \sum_{z} \sum_{a} X_{za} \right) \right) \quad \forall f
\end{align*}
\]
\[ D = \sum_g D_g \]  

\[ F = \sum_g F_g \]  

\[ Q = \sum_g Q_g \]  

\[ \sum_g \sum_z \eta_{gz} - 1 \leq \sum_g \sum_f \sum_z C_{fk} X_{fgz} \leq \sum_g \sum_z \eta_{gz} \quad \forall g, z \]  

\[ \sum_g \sum_z \eta_{gz} - 1 \leq \sum_g \sum_f \sum_z W_{fk} X_{fgz} \leq \sum_g \sum_z \eta_{gz} \quad \forall g, z \]  

\[ X_{fgz}, \eta_{gz} \geq 0 \quad \forall f, \forall g, \forall z \]  

\[ f \in [1, F] \quad g \in [1, G] \quad z \in [1, Z] \]  

\[ k \in [1, K] \quad q \in [1, Q] \]  

So the notations for first level are coming as:

- \( X_{fgz} \): The amount of \( f_{th} \) material produced by \( g_{th} \) supplier for delivering to \( z_{th} \) supplier.

- \( X_{fgz} \): The amount of \( f_{th} \) material produced by \( k_{th} \) team works in \( g_{th} \) supplier for delivering to \( z_{th} \) supplier.

- \( \alpha_{fk} \): The expected time for producing \( f_{th} \) material by \( k_{th} \) team works at \( g_{th} \) supplier.

- \( T_{kg} \): Total available time for \( k_{th} \) team works in \( g_{th} \) supplier.
• $X_{fgz}$: The amount of $f_{ih}$ material produced by $q_{th}$ machines at $g_{th}$ supplier for delivering to $z_{th}$ supplier.

• $t_{fgz}$: The expected time for producing $f_{ih}$ material by $q_{th}$ machines at $g_{th}$ supplier for delivering to $z_{th}$ supplier.

• $\delta_{qs}$: Probability of $q_{th}$ machines at $g_{th}$ supplier working properly at expected time.

• $S_{fgz}$: Setup time for $q_{th}$ machine at $g_{th}$ supplier for producing $f_{ih}$ material.

• $T_{qs}$: Total available time for $q_{th}$ machine at $g_{th}$ supplier.

• $r_{fb}$: Coefficient of $f_{ih}$ material for available $b_{ih}$ product at BOM.

• $d_{kg}^{-}$: Negative deviation variable of available time for $k_{th}$ team at $g_{th}$ supplier.

• $d_{kg}^{+}$: Positive deviation variable of available time for $k_{th}$ team at $g_{th}$ supplier.

• $d_{qs}^{-}$: Negative deviation variable of available time for $q_{th}$ machine at $g_{th}$ supplier.

• $d_{qs}^{+}$: Positive deviation variable of available time for $q_{th}$ machine at $g_{th}$ supplier.

• $TR_{gz}$: Shipment cost for $f_{ih}$ material of $g_{th}$ supplier to $z_{th}$ supplier.

• $\eta_{gz}$: Quantity of expected vehicles needs to ship the $f_{ih}$ material from $g_{th}$ supplier to $z_{th}$ supplier.

• $PC_{fg}$: Production Cost of $f_{ih}$ material at $g_{th}$ supplier.

• $C_{fg}$: Volume of $f_{ih}$ material at $g_{th}$ supplier.

• $W_{fg}$: Weight of $f_{ih}$ material at $g_{th}$ supplier.

• $V_{gz}$: Volume capacity of vehicle.
• \( V_{g}^{e} \): Weight capacity of vehicle.

The first function is minimizing the total costs of first level supplier. First part of functions is transportations cost of \( f^{th} \) materials ship to second level supplier. The second part of function shows the production costs at \( g^{th} \) supplier for \( f^{th} \) materials. The third part describes the amount of vehicles needed. This part is satisfied by constraint number 9 and 10. The second function is to minimizing the available time deviations for \( q^{th} \) machines and \( k^{th} \) teams. Constraints (2, 12, and 23) are team work flexibility, and it means multiplying quantity of \( f^{th} \) material at \( k^{th} \) group in \( g^{th} \) supplier by expected time for the same materials. Constraints (3 and 13) are machine flexibility, the ability of machines to adopt them for producing the new materials or products. Constraints (4, 5, 14, 15 and 22) are showing the expected demand goal that should be satisfied, and ensure that the model try to achieve its goal. Constraints (9 and 10) are determining the quantity of vehicles needed. These constraints could be seen in the next level models too.

The next step is to propose the second level model. The result that comes out of this model is to use for the next supplier. According to result obtained from previous model, we will step to prepare requirements of the next level.

So the model is coming like this:

\[
\begin{align*}
\text{Min} & \quad z = \sum_{z} \sum_{b} \sum_{a} TR_{za} X_{bza} + \sum_{z} \sum_{b} \sum_{a} PC_{bc} X_{bca} + \sum_{g} \sum_{a} \eta_{ac} TR_{ca} \\
\text{Min} & \quad \sum_{e} (d_{cz}^- + d_{cz}^+) \sum_{e} (d_{cz}^- + d_{cz}^+) \quad \forall z
\end{align*}
\]  \quad (11)
\[ \sum_{b} \left( \sum_{a} X_{bza} \right) \alpha_{bz e} + d_{ez}^{-} - d_{ez}^{+} = T_{ez} \quad \forall e, \forall z \quad (12) \]

\[ \sum_{b} \left( \sum_{a} X_{bza} \right) t_{bze} \delta_{ez} + S_{bz e} \right) + d_{ez}^{-} - d_{ez}^{+} = T_{ez} \quad \forall c, \forall z \quad (13) \]

\[ \sum_{a} \sum_{z} \left( \sum_{b} \left( X_{bza} + X_{bza} \right) \right) = 0.5 \sum_{a} \sum_{z} \sum_{b} X_{bza} \quad \forall e, \forall c \quad (14) \]

\[ \sum_{a} \sum_{z} X_{bza} = \sum_{i} \left( r_{bi} \cdot \left( \sum_{j} \sum_{a} \sum_{h} X_{ijah} \right) \right) \quad \forall b \quad (15) \]

\[ E = \sum_{z} E_{z} \quad (16) \]

\[ B = \sum_{z} B_{z} \quad (17) \]

\[ C = \sum_{z} C_{z} \quad (18) \]

\[ \sum_{z} \sum_{a} \eta_{za} - 1 \leq \sum_{z} \sum_{a} \sum_{g} C_{bza} \cdot X_{bza} \leq \sum_{z} \sum_{a} \sum_{g} \eta_{za} \quad \forall z, a \quad (19) \]

\[ \sum_{z} \sum_{a} \eta_{za} - 1 \leq \sum_{z} \sum_{a} \sum_{g} W_{bza} \cdot X_{bza} \leq \sum_{z} \sum_{a} \sum_{g} \eta_{za} \quad \forall z, a \quad (20) \]

\[ X_{bza}, \eta_{za} \geq 0 \quad \forall b, \forall z, \forall a \]

Parameters for second model are as below:
• $X_{bca}$: The amount of $b_{th}$ material produced by $z_{th}$ supplier for delivering to $a_{th}$ producer.

• $X_{beza}$: The amount of $b_{th}$ material produced by $e_{th}$ team works in $z_{th}$ supplier for delivering to $a_{th}$ producer.

• $\alpha_{bez}$: The expected time for producing $b_{th}$ material by $e_{th}$ team works at $a_{th}$ producer.

• $T_{ez}$: Total available time for $e_{th}$ team works in $z_{th}$ supplier.

• $X_{beza}$: The amount of $b_{th}$ material produced by $c_{th}$ machines at $z_{th}$ supplier for delivering to $a_{th}$ producer.

• $t_{bcz}$: The expected time for producing $b_{th}$ material by $c_{th}$ machines at $z_{th}$ supplier for delivering to $a_{th}$ producer.

• $\delta_{cz}$: Probability of $c_{th}$ machines at $z_{th}$ supplier working properly at expected time.

• $S_{bcz}$: Setup time for $c_{th}$ machine at $z_{th}$ supplier for producing $b_{th}$ material.

• $T_{cz}$: Total available time for $c_{th}$ machine at $z_{th}$ supplier.

• $r_{bi}$: Coefficient of $b_{th}$ material for available $i_{th}$ product at BOM.

• $d_{ez}^-$: Negative deviation variable of available time for $e_{th}$ team work at $g_{th}$ supplier.

• $d_{ez}^+$: Positive deviation variable of available time for $e_{th}$ team work at $g_{th}$ supplier.

• $d_{cz}^-$: Negative deviation variable of available time for $c_{th}$ machine at $z_{th}$ supplier.

• $d_{cz}^+$: Positive deviation variable of available time for $c_{th}$ machine at $z_{th}$ supplier.

• $TR_{za}$: Shipment cost for $b^{th}$ material of $z^{th}$ supplier to $a^{th}$ factory.
• \( \eta_{ca} \): Quantity of expected vehicles needs to ship the \( b^{th} \) material from \( z^{th} \) supplier to \( a^{th} \) factory.

• \( PC_{bz} \): Production Cost of \( b^{th} \) material at \( z^{th} \) supplier.

• \( C_{bz} \): Volume of \( b^{th} \) material at \( z^{th} \) supplier.

• \( W_{bz} \): Weight of \( b^{th} \) material at \( z^{th} \) supplier.

• \( V_{\gamma}^g \): Volume capacity of vehicle.

• \( V_{\omega}^g \): Weight capacity of vehicle.

And for the third level we have:

\[
Minz = \sum_{a} \sum_{i} \sum_{j} \sum_{h} TR_{ah} X_{ijah} + \sum_{a} \sum_{i} \sum_{j} \sum_{h} PC_{ijah} X_{ijah} + \sum_{a} \sum_{h} \eta_{ah} TR_{ah}
\tag{21}
\]

\[
Minz = \sum_{h} (d_{ih}^- + d_{ih}^+) \cdot \sum_{a} (d_{ja}^- + d_{ja}^+) \quad \forall i, \forall j
\]

S.t. \[
\sum_{a} \sum_{h} \sum_{j} X_{ijah} + \sum_{h} (d_{ja}^- + d_{ja}^+) = \sum_{h} D_{dh} \quad \forall i
\tag{22}
\]

\[
\sum_{i} \left( \sum_{h} X_{ijah} \right) \alpha_{ja} + d_{ja}^- - d_{ja}^+ = T_{ja} \quad \forall j, \forall a
\tag{23}
\]

\[
J = \sum_{a} J_a
\tag{24}
\]

\[
I = \sum_{a} I_a
\tag{25}
\]

\[
\sum_{a} \sum_{h} \eta_{ah} - 1 \leq \frac{\sum_{a} \sum_{i} \sum_{j} \sum_{h} C_{ijah} X_{ijah}}{V_{\gamma}^g} \leq \sum_{a} \sum_{h} \eta_{ah} \quad \forall a, j
\tag{26}
\]
\[
\sum_a \sum_h \eta_{ih} - 1 \leq \frac{\sum_a \sum_j \sum_h W_{ija} X_{ijah}}{V_{ah}} \leq \sum_a \sum_h \eta_{ah} \quad \forall a, j
\] (27)

\[X_{ijah} \geq 0 \quad \forall i, \forall j, \forall a, \forall h\]

\[X_{iah} \geq 0 \quad \forall i, \forall a, \forall h\]

\[i \in [1, I] \quad a \in [1, A]\]

\[h \in [1, H] \quad j \in [1, J]\]

- \(X_{ijah}\): The amount of \(i_{th}\) material produced by \(j_{th}\) team work at \(a_{th}\) factory for delivering to \(h_{th}\) market.

- \(\alpha_{ija}\): The expected time for producing \(i_{th}\) material by \(j_{th}\) team works at \(a_{th}\) factory.

- \(T_{ja}\): Total available time for \(j_{th}\) team works in \(a_{th}\) factory.

- \(d_{ih}^{-}\): Negative deviation variable of demand for \(i_{th}\) product at \(h_{th}\) market.

- \(d_{ih}^{+}\): Positive deviation variable of demand for \(i_{th}\) product at \(h_{th}\) market.

- \(d_{ja}^{-}\): Negative deviation variable of available time for \(i_{th}\) product at \(h_{th}\) market.

- \(d_{ja}^{+}\): Positive deviation variable of available time for \(i_{th}\) product at \(h_{th}\) market.

- \(TR_{za}\): Shipment cost for \(b^{th}\) material of \(z^{th}\) supplier to \(a^{th}\) factory.

- \(\eta_{za}\): Quantity of expected vehicles needs to ship the \(b^{th}\) material from \(z^{th}\) supplier to \(a^{th}\) supplier.

- \(PC_{ijah}\): Production Cost of \(i^{th}\) material at \(j^{th}\) team work of \(a^{th}\) factory to ship in \(h^{th}\) market.
• $C_{ijah}$: Volume of $i^{th}$ product at $j^{th}$ team work of $a^{th}$ factory to ship in $h^{th}$ market.

• $W_{ijah}$: Weight of $i^{th}$ product at $j^{th}$ team work of $a^{th}$ factory to ship in $h^{th}$ market.

• $V_{ah}^T$: Volume capacity of vehicle.

• $V_{ah}^w$: Weight capacity of vehicle.

### 4- Numerical Example and Results

In this section, the model is run. 5 cases are implemented for this study. Table 1 shows the data we have used.

**Table 1 about here**

For simplifying the model, we suppose that we have one kind of vehicle with a constant capacity.

The table 2 shows the parameters applied in this study.

**Table 2 about here**

Table 3 shows the results of every 5 cases in this study.

**Table 3 about here**

For solving the problem of our study, we used LINGO 9.0 package. As it is clear in table, when we increase the nodes on SCN, the cost increases too. Also the time processing is increased. In this study, goals deviations for all cases were 0, and it means that the model achieved its goal and has not surplus and lack deviations.

### 5- Conclusion
In this study, we develop the mathematical model of supply chain network, and implemented 5 cases studies. First, we review the relevant literature of supply chain management and models applied to optimize materials flow, and then we introduce the Aprile (2005) model. After that, we proposed our developed model. The proposed model is like a DSS tool for managers to help them make a better decision. The results obtained in this study, showed that if nodes on SCN increase, the cost will increase too. The implementation of the proposed model of this paper has been used for a case study of real-world problem in an auto-part maker. It is so important for managers know how to produce and supply materials that optimize their cost and revenue. Some recommendations for future studies are:

1- Applying Meta heuristic methods like GA, ABC, SA, AC and compare the results to our model.

2- Put some variables like risk, to improve the model.

3- Using fuzzy sets theory to make decision in uncertainly environment and develop the model.

4- For future study, suggested bullwhip effect to be entered to model.

References


Figure 1. Supply chain of a company (Poirier 1999)

Fig. 1. Major purchasing processes.
Figure 2. Principle process of procurement (Aissaoui et al. 2007)

Figure 3. Supply chain network

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<th>Market</th>
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Table 1. Data for every case
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</thead>
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<td>100~10,000</td>
<td>Ton</td>
</tr>
<tr>
<td>2</td>
<td>Raw Material of Product</td>
<td>0.09~0.3</td>
<td>Ton</td>
</tr>
<tr>
<td>3</td>
<td>Unit Production Cost of Raw Material by Suppliers</td>
<td>60~120</td>
<td>Dollar</td>
</tr>
<tr>
<td>4</td>
<td>Unit Manufacture Cost of Product by Factory</td>
<td>5~12</td>
<td>Dollar</td>
</tr>
<tr>
<td>5</td>
<td>Unit Transportations from Suppliers to Factory</td>
<td>0.9~1.6</td>
<td>Dollar</td>
</tr>
<tr>
<td>6</td>
<td>Unit Transportation Cost of Product that Transported from Factory to Market</td>
<td>0.6~7.5</td>
<td>Dollar</td>
</tr>
<tr>
<td>7</td>
<td>Unit Transportation Cost of Product that Transported from Distribution Center to Retailer</td>
<td>1~8</td>
<td>Dollar</td>
</tr>
<tr>
<td>8</td>
<td>Weight capacity of vehicles</td>
<td>15~30</td>
<td>Ton</td>
</tr>
<tr>
<td>9</td>
<td>Setup Time for Machines in Suppliers and Factory</td>
<td>0.3~10</td>
<td>Min</td>
</tr>
</tbody>
</table>

Table 2. Parameters used in this paper
<table>
<thead>
<tr>
<th>Cases</th>
<th>Total Cost($)</th>
<th>Quantity of Materials</th>
<th>Quantity of Products</th>
<th>Quantity of Vehicles</th>
<th>Time process</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20000</td>
<td>Sup1 (1000)</td>
<td>1530</td>
<td>S₁ to S₂ (3)</td>
<td>2 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sup2(890)</td>
<td></td>
<td>S₂ to F (2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>F to H (1)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28000</td>
<td>Sup1 (1981)</td>
<td>2000</td>
<td>S₁ to S₂ (5)</td>
<td>4 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sup2(1130)</td>
<td></td>
<td>S₂ to F (3)</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td>F to H (3)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>46000</td>
<td>Sup1 (2400)</td>
<td>5147</td>
<td>S₁ to S₂ (7)</td>
<td>5 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sup2(2600)</td>
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<td>S₂ to F (4)</td>
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<td>F to H (3)</td>
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</tr>
<tr>
<td>4</td>
<td>60123</td>
<td>Sup1 (3200)</td>
<td>5800</td>
<td>S₁ to S₂ (9)</td>
<td>7 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sup2(3100)</td>
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<td>S₂ to F (8)</td>
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<td>F to H (5)</td>
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<tr>
<td>5</td>
<td>97300</td>
<td>Sup1 (4490)</td>
<td>4900</td>
<td>S₁ to S₂ (9)</td>
<td>10 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sup2(1800)</td>
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<td>S₂ to F (5)</td>
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<td>F to H (4)</td>
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</table>

Table 3. Results of research